SIMULATION OF OPINION STRUCTURES IN SOCIAL SYSTEMS

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1. Introduction

Inspired by the obvious analogy in the behaviour of social systems consisting of many inhabitants and many particle physical systems, the authors propose a quite simple model to discuss the formation of structures in social systems.

The basic feature of our proposal arises from the theory of reaction-diffusion systems. Social ingredients are considered in a simple way, justified only by common sense, but the model is open for an improved ansatz resulting from quantitative sociology.

One of the main premises of the theory of self-organization means that complex systems are constructed by a large number of subsystems. These subsystems have their own dynamics, but the structures obtained on a macroscopic level arises from the interactions between the subsystems.

Starting from this idea, we consider a system divided into a number of subsystems; this means the system consists of many boxes. The boxes ensure a spatial inhomogeneous structure of the system, they act as a grid which includes the <u>local</u> effects differing from box to box.

The <u>reaction</u> means a change of the opinion of some individuals, it is not connected with any spatial changes. These reactions are determined mainly by the "opinion climate" in the given subsystem.

The <u>diffusion</u> means the spatial dispersal (migration) of individuals with a certain opinion, which does not change the opinions.

Additionally, we consider the influence of external and internal "fields", this means e.g. an external political pressure, or the common opinion, spread by papers and TV.

2. Model of Opinion Formation

2.1. Basic Assumptions

(i) Society

Our systems consists of N individuals which are spatially distributed to z boxes indicated by k=1,2,...,z, wich means a spatial coordinate. Since the total number of individuals is conserved, it yields:

$$N_{total} = \sum_{k=1}^{z} N^k = \text{const.} \qquad (2.1)$$

The volume V^k of the boxes is assumed to be equal.

(ii) Opinion Scale

Every individual has a special state called its opinion (with respect to a definite aspect or problem). The different opinions get (discrete) numbers in order to divide between them.

We assume an "order" of the opinions, expressed by their numbers. Introducing a scale

$$\underline{M} = \{-M_1,\dots,-2,-1,0,1,\dots,m,\dots,M_2\} \qquad (2.2)$$

which means that an opinion with a very negative number is located at the

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extreme left spectrum of opinions, and an opinion with a large positive number on the right side of the scale. Opinions with small (positive or negative) numbers arise from the indifferent middle region of opinions.

(iii) Socioconfiguration

The socioconfiguration is assumed to be the appropriate macrovariable to express the state of the considered "society" (Weidlich and Haag, 1983). Here it means the distribution of the individuals with respect to their <u>location</u> (box index k) and to their <u>opinion</u> (opinion index m), characterized by the discrete vector

$$\underline{N}^k = \{\dots(N_m^1 N_m^2 \dots N_m^k \dots N_m^z)\dots\} \qquad m = -M_1,\dots,M_2 \qquad (2.3)$$

N_m^k means the number of individuals with opinion m in box k=1,...,z.

A change of the socioconfiguration (2.3) includes two different processes: (1) a change of the individual opinions (on a microscopic level), (2) a change of the location of the individuals by means of migration. For these events we now need certain assumptions.

2.2. Change of Opinions

The reasons for a change of opinions of some individuals are intrinsically determined by sociological, psychological, political a.o. influences. That means their analysis is a complex field for different sciences and not the task of this short paper.

We choose here some simple relations, confirmed by common sense. Considering two different cases:

(i) Changes of Opinion by Individual Decisions

The individual changes its opinion without direct interactions with other individuals, namely

* spontanously, that means without any reason, which can be detected from outside
* by influence of an "external field" E, which affects on the whole society via a political or ethical "pressure", a war danger, an ecological threatening and others. This field supports a certain opinion of the scale, that means $E \in \underline{M}$.
* by influence of an "internal field" Q, created by the opinions of the individuals of the society (common opinion). This field acts e.g. via newspapers, TV and means a <u>indirect</u> interaction of the individuals.

The change of the individual opinion without direct interaction can be formally expressed by the reaction

$$X_m^k \xrightarrow{a(j|m)} X_j^k \qquad (2.4)$$

where a(j|m) is the individual transition rate for the change of opinion from m to j. It yields a(m|m) = 0. We make the following ansatz:

$$a(j|m) = C_m \exp\{-\alpha|m-j| - \beta(|j-E|-|m-E|) - \gamma(|j-Q|-|m-Q|)\} \qquad (2.5)$$

Eq. (2.5) means that the probability of change of the opinion decreases with the difference between the old and the new opinion, it decreases further, if the difference to the external field E and to the internal field Q is increasing.

The influence of the three terms in eq. (2.5) are weighted by small constants α, β, γ which are normally positive values. But for individuals with extreme opinions they could be also negative, that means this individual prefers an opinion <u>opposite</u> to the common opinion Q, for instance.

The internal field Q results from the weighted opinions of all individuals, like

$$Q = (1/z)\sum_{k=1}^{z}\psi^k; \qquad Q^k = \sum_{m=-M_1}^{M_2} \tau_m\, m\, N_m^k \Big/ \sum_{m=-M_1}^{M_2} N_m^k \qquad (2.6)$$

τ_m is the weighting parameter, Q^k is the internal field of box k, that means of the direct surroundings of a certain individuum in box k. Q is the mean value of all internal box fields.

The prefactor C_m in (2.5) characterizes the mobility of opinion change, that means the general readiness of an individual to change the opinion at all.

(ii) Changes of Opinion by Direct Interactions

The second case considered here includes the changes of opinions if one individual convinces another (direct interaction). This can be described by the reaction scheme:

$$X_m + X_j \xrightarrow{b(j|m)} 2 X_j \qquad (2.7)$$

where $b(j|m)$ is the individual transition rate for a change of opinion from m to j. Eq. (2.7) includes only interactions between two individuals at the same time. It is assumed that only one individual changes its opinion, that means the other has convinced it. For $b(j|m)$ we make the following ansatz:

$$b(j|m) = C_m \exp\{-\delta(N_m^k - N_j^k)\} \qquad (2.8)$$

Eq. (2.8) means that the change of opinions in general follows the opinion of the majority. The small parameter δ is positive, but it could be also negative for individuals which are always in opposition to the majority.

The reaction (2.8) reflects further that only individuals interact which are in the same box k, this underline the local effect of interactions.

2.3. Spatial Changes of Individuals (Migration)

The individuals of our system are able to leave their box and to migrate into another subsystem which must be an adjecting box. Jumps over large distances are not considered here (cf. e.g. Weidlich and Haag, 1988).

The migration is described in terms of a diffusion process, which can be expressed by the reaction scheme

$$X_m^k \xrightarrow{d_m^{nk}} X_m^n \qquad (2.9)$$

n and k must be adjecting boxes, d_m^{nk} is the effective diffusion coefficient for species m between k and n.

It has been pointed out (e.g. by Skellam 1973, Malchow 1988) that biological species are not simple diffusers at all; we obtain neutral Fickian Diffusion as well as repulsive dispersal or attractive dispersal.

We propose the following simple ansatz:

$$d_m^{nk} = D_m \exp\{-\varepsilon |m| N_m^k\} \qquad (2.10)$$

The dispersal from a given subsystem is determined by a spatial mobility D_m, which depend on the opinion m. Further we consider a special kind of concentration dependence of d_m^{nk} via the exponent: for $\varepsilon > 0$ we have a repulsive dispersal, the individuals spread out over the whole system. This effect decreases with the deviation of the opinion from the "0"-opinion. The so-called "non-politicals" are always equal distributed, since $d_0^{nk} = D_0 =$ const. But individuals with extreme opinions, say with large $|m|$, mostly stay together in one box (e.g. an "autonomic scene"), therefore their diffusion coefficient is rather small, only if the number of like-minded people is rather high in the given subsystem.

We note that the choice of the transition rates presented here does not detract from the general theory, which is open to consider also more

proved relations coming from quantitative sociology. The parameters α, β, γ, δ, ε are free, their variations may include a large variety of different cases. The discussion in Chapter 4 shows only some simple applications.

3. Kinetic Description of the Evolution of the Socioconfiguration

3.1. Stochastic Approach – The Multivariate Master Equation

From a stochastic point of view every possible socioconfiguration $\underline{N}$ exists with a certain probability, defined by $P(\underline{N},t)$. This probability may change via "reactions", say change of opinions, and via "diffusion", say migration processes. Their time-dependent evoluions is described by the so called multivariate master equation (Gardiner, 1983), which reads:

$$\frac{\partial P(\underline{N}^*,t)}{\partial t} = \sum_m \sum_{\substack{i,j \\ (i \neq j)}} \{ w_{mi}\, P(N_m^i+1, N_m^j-1, \underline{N}^{**},t) - w_{mi}\, P(\underline{N}^*,t) \}$$
$$+ \sum_m \sum_{\substack{k \\ i,j \text{ adjecting boxes}}} \{ w_{rw}\, P(N_m^k-1, N_j^k+1, \underline{N}^{**},t) - w_{rw}\, P(\underline{N}^*,t) \} \qquad (3.1)$$

The value $\underline{N}^{**}$ expresses those elements of the vector (2.3) which have not changed, the changed elements are written explicitely here.

w_{mi} and w_{rw} are the transition probabilities for the migration and for the change of opinions. With respect to the individual transition rates introduced in Chapter 2 they explicitely read as follows:

$$w_{mi}(N_m^k-1 | N_m^k) = \Sigma_j\, d_m^{jk}\, N_m^k/V^* \equiv w_{mi}^-(N_m^k) \qquad (3.2)$$

$$w_{mi}(N_m^k+1 | N_m^k) = \Sigma_j\, d_m^{kj}\, N_m^j/V^* \equiv w_{mi}^+(N_m^k) = \Sigma_j\, w_{mi}^-(N_m^j) \qquad (3.3)$$

$$w_{rw}(N_m^k+1 | N_m^k) = \Sigma_j\, \{a(m|j) + b(m|j)\, N_m^k\}\, N_j^k/V^* \qquad (3.4)$$

$$w_{rw}(N_m^k-1 | N_m^k) = \Sigma_j\, \{a(j|m) + b(j|m)\, N_j^k\}\, N_m^k/V^* \qquad (3.5)$$

It depends mainly on the relation between the coefficients of reaction, a and b, and the coefficients of migration, d, whether the change of opinions or the migration processes influence the evolution of the socioconfiguration more considerable.

The multivariate master equation allows the solution by means of a stochastic computer simulation of the kinetics which includes the probabilistic aspect of the evolution (cf. e.g. Schweitzer et.al., 1988). But we restrict ourselves here to the discussion of the deterministic case.

3.2. Deterministic Approach – The Mean Values Equations

By means of a known projector formalism (see e.g. Ebeling and Feistel, 1982) we are able to derive from the master equation the mean values equations for the deterministic evolution of the socioconfiguration.

As the result we arrive finally at the following set of coupled equations:

$$\langle \dot{N}_{mk} \rangle = (1/V^*) \sum_{j=-M_1}^{M_2} [\langle a(j|m)\, N_j^k - a(m|j)\, N_m^k \rangle + \langle \{b(m|j) - b(j|m)\}\, N_m^k N_j^k \rangle]$$
$$+ (1/V^*) \sum_{n=1}^{NN} \{ \langle d_m^{kn}\, N_m^n \rangle - \langle d_m^{nk}\, N_m^k \rangle \} \qquad (3.6)$$

NN means the sum over all nearest neighbours of the box.

Introducing now the net transition rates and abbreviations:

$$A_{mj} = -a(m|j)/V^* \quad \text{for } j \neq m; \quad A_{mm} = \Sigma_j\, a(j|m)/V^*$$
$$B_{mj} = \{b(m|j) - b(j|m)\}/V^* = -B_{jm}$$
$$D_m^{kn} = d_m^{kn}/V^* \qquad (3.7)$$

we find the deterministic equation in the compact form:

$$\langle \dot{N}_m^k \rangle = \Sigma_j \langle A_{mj} N_j^k \rangle + \langle B_{mj} N_m^k N_j^k \rangle + \Sigma_n \langle D_m^{kn} N_m^n - D_m^{nk} N_m^k \rangle \qquad (3.8)$$

Because of the conservation of the total number of individuals it yields

$$\sum_{k=1}^{z} \sum_{m=-M_1}^{M_2} \langle \dot{N}_m^k \rangle = 0 \qquad (3.9)$$

resulting in:

$$\sum_{j,m=-M_1}^{M_2} A_{mj} = 0 \;; \quad \sum_{j,m=-M_1}^{M_2} B_{mj} = 0 \;; \quad \frac{1}{2} \sum_{k=1}^{z} \langle D_m^{kn} N_m^n - D_m^{nk} N_m^k \rangle = 0 \qquad (3.10)$$

4. Numerical Examples and Discussion

4.1. Investigation of Stationary States of the Homogeneous System

(a) Following the formalism of Shapiro and Horn, 1979, a detailed analysis of the system of equations has been carried out for the homogeneous case (that means without spatial differences). The procedure shows, that multiple steady states of the homogeneous system exist only if we include direct interactions between the individuals in our description. Considering only individual decisions, we find only one stable state. This result holds independently on the specific transition rates.

(b) The number of stationary states of the homogeneous system depends on whether we consider direct interactions between all opinions or only between neighboring opinions (keeping in mind, that completely opposite groups will not speak with each other).
We discuss the example of three different opinions $\underline{M} = \{-1,0,+1\}$.

1. Assuming first only individual decisions, the stable state in the absence of an external field is given by

$$N_{-1} = N_{+1} = N_0 = N_{total}/3 \qquad (4.1)$$

   A remarkable influence of E may shift the ratio in the direction of E.
2. Considering now additionally to (1.) interactions between all opinions, we find, that the former stable state (4.1) becomes an instable node. Additionally, an (instable) saddle point is obtained. Further, we observe three new stable states, characterized by at that time one dominating opinion.
3. Considering only individual decisions and interactions between neighbouring opinions, we find the same instable node and the same saddle point as discussed in (2.), but the three stable states are now different: two stable states are characterized by a coexistence of two contradicting opinions, while the third stable state means one dominating opinion again. (compare Fig. 1)

4.2. Numerical Simulations of the Dynamic Behaviour of the Socioconfiguration

As mentioned above, one way to study the kinetics of the socioconfiguration is the stochastic simulation in order to solve the master equation. The other way is the numerical investigation of the system of coupled differential equations. We restrict ourselfes here to the latter case.

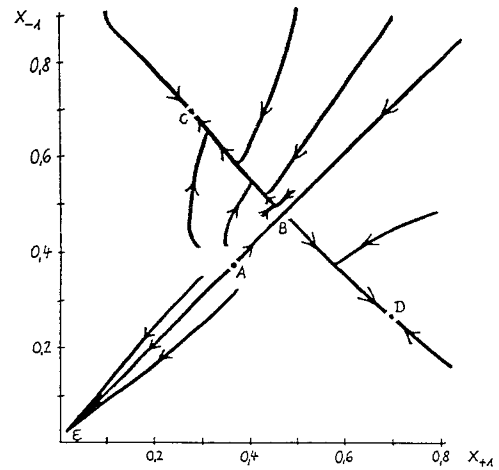

Fig. 1: Phase trajectories and stationary states in a homogeneous system with direct interactions between individuals with neighboring opinions $X_{-1} = N_{-1}/N_{total}$; $X_{+1} = N_{+1}/N_{total}$, $X_0 = 1 - X_{-1} - X_{+1}$
A: instable node; B: saddle point; C, D: stable coexistence between two opinions; E: stable state with one dominating opinion (0)
parameters: $\alpha=0.5$; $\beta=0.5$; $\gamma=0$; $\delta=0.01$; $E=+2$; $C_m=1$

For z boxes and M opinions we have, with respect to the boundary condition (2.1), z*M-1 coupled equations. Our numerical examples are carried out for a square unit of 7x7 boxes with periodic boundary conditions and a spectrum of opinions $\underline{M} = \{-2,-1,0,+1,+2\}$, that means 250 equations at each time step.

(a) Spread of one extreme opinion
This example has an analogy to the colonisation of an uninhabited area. We consider a central box with 10.000 inhabitants at time t=0. They have the opinion m=-2, that means, they are very settled and have a very small migration (cf. eq. (2.10)).
Since all other boxes are uninhabited and the concentration of the settled inhabitants is very high, a certain "pressure" exists, which forces the migration: E = (0).

1. period: In the mother box spontanously a new opinion is created: to migrate. This means a transition from opinion (-2) to (0), because the (0)-opinion has a large diffusion coefficient
2. period: Some of the inhabitants with opinion (0) migrate into surrounding subsystems, but they rest only in boxes where the migration from the origin box is not so remarkable. In boxes in the vicinity of the mother box the concentration of inhabitants is not so high.
3. period: The inhabitants which have been rested in the new subsystems, change their opinion again from (0) to (-2), they become settled. This is caused by their "cultural memory", expressed by the internal field Q, which is (in the beginning of the simulation) very close to (-2). If the concentration of inhabitants in the new subsystem increases again, a new cycle occurs.

The result of this dynamic behaviour are periodic rings in a certain distance from the mother box. These rings are characterized by a significant high number of inhabitants, divided by boxes with a lower density.
In Fig. 2 this fact is plotted for the case of a linear box system. The mother box is k=25, in a certain distance from the source (about 10 boxes) we observe periodic structures.

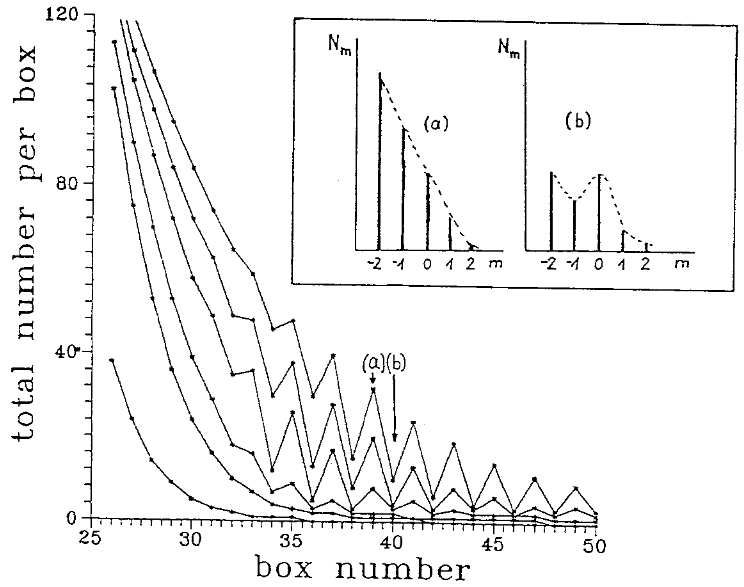

Fig. 2: Evolution of the total number per box in a chain of boxes for different time steps; initial state: N_{-2} = 10.000 in box 25
The sketches (a) and (b) give the opinion distributions for the boxes with high (a) and low (b) particle density
parameters: α=0.2; β=0.1; γ=0.3; δ=0.01; ε=0.7, C_m=D_m=1, E=(0)

We note the fact, that the distribution of opinions in boxes with lower density of inhabitants is completely different from these with higher density, which is sketched also in Fig. 2. We find either an unimodal or a bimodal distribution of opinions. The difference results mainly from the inhabitants with opinion (-2), these with opinion (0) are rather equal distributed.

Asymptotically all inhomogenities in the system are damped out, that means we have in all subsystems the same unimodal distribution where the internal field Q has shifted from (-2) to (-1).

(b) Spread of two extreme opinions from different sources
This example which cannot be discussed here because of the limited place, leads to interesting patterns. We observe different regions with unimodal distributions of opinions, which are divided by border regions with a bimodal distribution. The border regions are not fixed, they change their position like fronts, which come to rest in the stationary state.

Finally, we note, that the model developed in this short paper, includes a variety of different cases, which should be simulated by means of different sets of parameters. The given examples mean only an introducting demonstration.

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